Two well-insulated rigid tanks A and B have the same volume (V_A=V_B=1 m^3) and both are filled with air with the following thermodynamic states:

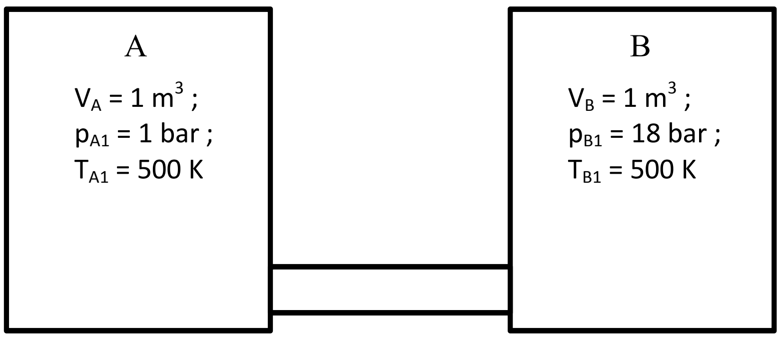
Tank A: p_A = 1 bar ; T_A = 500 K

Tank B: p_B = 18 bar ; T_B = 500 K

Now these two tanks are connected with each other through a pipe, so that air from tank A and tank B can mix with each other and reach the same pressure in the end. During this mixing process, no heat is transferred.

Determine the entropy production after this mixing process.

Assume that in this case, air is an ideal gas and its specific gas constant is R=287 J/kg·K



Solution:

Index 1: state before mixing

Index 2: state after mixing

At first, we calculate the mass of gas in both tanks before mixing:

$$m_A = \frac{p_{A1} \cdot V_{A1}}{R \cdot T_{A1}} = \frac{1 \cdot 10^5 Pa \cdot 1m^3}{287\ \mathrm{J/kg \cdot K} \cdot 500K} = 0.697\ kg$$

$$m_B = \frac{p_{B1} \cdot V_{B1}}{R \cdot T_{B1}} = \frac{18 \cdot 10^5 Pa \cdot 1m^3}{287\ \mathrm{J/kg \cdot K} \cdot 500K} = 12.54\ kg$$

In order to determine the entropy production, pressure and temperature p_2 and T_2 after mixing should also be obtained.

The boundary of the control volume chosen in this case encloses both tanks, tank A and tank B. Since both tanks are well-insulated and rigid, there are no heat transfer and work for volume change. Hence we get the simplified equation of the 1st Law of thermodynamics for closed system and the final temperature T_2:

$$\Delta U=\Delta U_A+\Delta U_B=0$$

$$\rightarrow m_A \cdot c_v \cdot (T_2-T_A)+ m_B \cdot c_v \cdot (T_2-T_B)=0$$

$$\rightarrow T_2=T_A= T_B=500K$$

So the final pressure p_2 is:

$$p_2 = \frac{m_{tot} \cdot R \cdot T_2}{V_{tot}} = \frac{(m_A + m_B) \cdot R \cdot T_2}{V_A + V_B} = 9.5\ bar$$

Then the entropy production is:

$$\Delta S = \Delta S_A + \Delta S_B = -m_A \cdot R \cdot \ln\left(\frac{p_2}{p_{A1}}\right) + \left(-m_B \cdot R \cdot \ln\left(\frac{p_2}{p_{B1}}\right)\right)$$

$$\Rightarrow \Delta S = -0.697kg \cdot 287 \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}} \cdot \ln\left(\frac{9.5\ bar}{1\ bar}\right) + \left(-12.54kg \cdot 287 \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}} \cdot \ln\left(\frac{9.5\ bar}{18\ bar}\right)\right)$$
$$= 1.85\ kJ/K \quad (> 0)$$

We can obtain an important conclusion:

The entropy after gas mixing and expansion will be produced (entropy production >0) and therefore this process is irreversible.