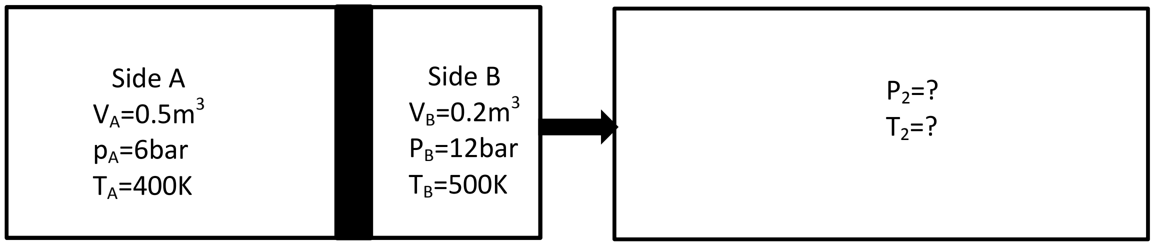
A well-insulated rigid tank is divided into part A and part B through a partition. Initially, side A of the tank contains $0.5m^3$ air at 6bar and 400K, and the other side B is filled with $0.2\ m^3$ air at 12bar and 500K. The partition is then removed, and the air mixes within the entire tank. Determine the final temperature of air.

Here the air is assumed to be ideal gas.

The specific gas constant for air is $R_{air}$=287 J/kg·K and the specific heat capacity at constant pressure $c_p$=1 J/g·K




Solution:

We first calculate the mass of air in each side of tank:

$$m_A = \frac{p_A \cdot V_A}{R_{air} \cdot T_A} = \frac{6 \cdot 10^5 Pa \cdot 0.5m^3}{287J/kg \cdot K \cdot 400K} = 2.613kg$$

$$m_B = \frac{p_B \cdot V_B}{R_{air} \cdot T_B} = \frac{12 \cdot 10^5 Pa \cdot 0.2m^3}{287J/kg \cdot K \cdot 500K} = 1.672kg$$

Since tank is rigid and its volume remains constant, there is no work for volume change. In addition, there is no other kind of work such as electrical work involves. → W=0

As the tank is well-insulated (adiabatic), no heat transfer occurs. → Q=0

According to the 1.Law of thermodynamics for closed system, the equation reduces to $\Delta U_{sys}=0$, that is, $\Delta U_A+\Delta U_B=0$.

Because the air is considered as ideal gas and we assume the final temperature $T_2$:

$$\Delta U_A=m_A \cdot c_v \cdot (T_2 - T_A) \quad \text{and} \quad \Delta U_B=m_B \cdot c_v \cdot (T_2 - T_B)$$

Therefore: $\Delta U_A+\Delta U_B= m_A \cdot c_v \cdot (T_2 - T_A)+ m_B \cdot c_v \cdot (T_2 - T_B)=0$

$$\Longrightarrow T_2 = \frac{m_A \cdot T_A + m_B \cdot T_B}{m_A + m_B} = \frac{2.613kg \cdot 400K + 1.672kg \cdot 500K}{2.613kg + 1.672kg} = 439.02K$$

Now we calculate the final pressure $p_2$ after removing the partition:

$$p_2 = \frac{m_2 \cdot R_{air} \cdot T_2}{V} = \frac{(m_A + m_B) \cdot R_{air} \cdot T_2}{V_A + V_B}$$
$$= \frac{(2.613 + 1.672)kg \cdot 287J/kg \cdot K \cdot 439.02K}{0.5m^3 + 0.2m^3} = 7.713bar$$

Since entropy is the property of state, we can calculate the entropy change of this system separately. That means,

$$\Delta S_{sys} = \Delta S_A + \Delta S_B$$

where

- $\Delta S_A$ is the entropy change of air in side A
- $\Delta S_B$ is the entropy change of air in side B

$$\Delta S_A = m_A \cdot \left(c_p \cdot ln\frac{T_2}{T_A} - R_{air} \cdot ln\frac{p_2}{p_A}\right)$$
$$= 2.613kg \cdot \left(1 \cdot 10^3 J/kg \cdot K \cdot ln\frac{439.02K}{400K} - 287J/kg \cdot K \cdot ln\frac{7.713bar}{6bar}\right)$$
$$= 54.88\,J/K$$

$$\Delta S_B = m_B \cdot \left(c_p \cdot ln\frac{T_2}{T_B} - R_{air} \cdot ln\frac{p_2}{p_B}\right)$$
$$= 1.672kg \cdot \left(1 \cdot 10^3 J/kg \cdot K \cdot ln\frac{439.02K}{500K} - 287J/kg \cdot K \cdot ln\frac{7.713bar}{12bar}\right)$$
$$= -5.366\,J/K$$

$$\Longrightarrow \Delta S = \Delta S_A + \Delta S_B = 54.88J/K - 5.366\,J/K = 49.514J/K$$

We notice that $\Delta S > 0$. That means, this process is irreversible which follows the 2.Law of thermodynamics.