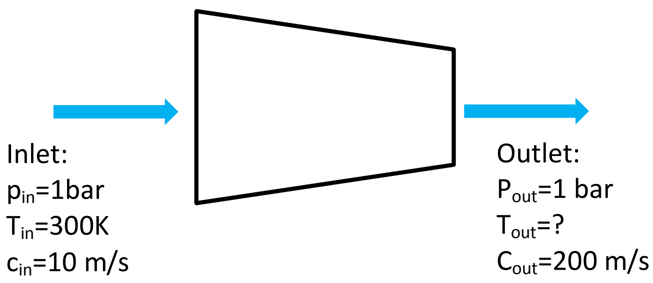
Air flows through an adiabatic and horizontal placed pipe with changeable cross section. At the inlet side air flows with velocity c_{in}=10 m/s under the pressure p_{in}=1bar and the temperature ϑ_{in}=25°C. At the outlet side air flows out of the pipe with velocity c_{out}=150 m/s under the pressure p_{in}=0.8bar.

1) Determine whether this pipe is nozzle or diffuser without calculating the ratio of cross section of outlet and inlet A_{out}/A_{in}
2) Determine now the ratio of cross section of outlet and inlet A_{out}/A_{in}

Air can be considered as perfect gas with c_p=1 J/K·g and R=0.29 J/K·g

Solution:



1)

The 1.Law of thermodynamics:

$$\sum_i \dot{Q}_i + \sum_j \dot{W}_j + \sum_k \dot{m}_k \cdot \left(h + \frac{c^2}{2} + g \cdot z\right)_k = \frac{d}{d\tau}(\sum_l m_l \cdot (\bar{u} + \bar{e}_{kin} + \bar{e}_{pot})_l)$$

Stationary → d/dτ=0

Adiabatic → ΣQ=0

Without input and output (work) → ΣW=0

Horizontal placed → no change in potential energy → Δ(g·z)=0

Then we can obtain the simplified 1.Law:

$$h_{in} + \frac{c_{in}^2}{2} = h_{out} + \frac{c_{out}^2}{2}$$

wobei $h_{in}=c_p \cdot T_{in}$ und $h_{out}=c_p \cdot T_{out}$

Therefore the temperature of air at the outlet side is:

$$T_{out} = T_{in} + \frac{c_{in}^2 - c_{out}^2}{2 \cdot c_p} = (25 + 273.15)K + \frac{(10m/s)^2 - (150m/s)^2}{2 \cdot 1000 J/K \cdot kg} = 286{,}95K$$

We now calculate the entropy change during this process:

$$\Delta s = c_p \cdot \ln\left(\frac{T_{out}}{T_{in}}\right) - R \cdot \ln\left(\frac{p_{out}}{p_{in}}\right)$$
$$= 1J/K \cdot g \cdot \ln\left(\frac{280.05K}{300K}\right) - 0.29J/K \cdot g \cdot \ln\left(\frac{1\ bar}{1.5\ bar}\right) = 0.049J/K \cdot g > 0$$

Hence it is Nozzle.

2)

In order to determine A_{out}/A_{in}, we need additional the law of conservation of mass and the ideal gas law:

$$\dot{m}_{in} = \dot{m}_{out} \Longrightarrow \rho_{in} \cdot c_{in} \cdot A_{in} = \rho_{out} \cdot c_{out} \cdot A_{out}$$

$$\frac{p_{in}}{\rho_{in}} = R \cdot T_{in} \quad und \quad \frac{p_{out}}{\rho_{out}} = R \cdot T_{out}$$

$$\Longrightarrow \frac{A_{in}}{A_{out}} = \left(\frac{p_{out}}{p_{in}}\right) \cdot \left(\frac{T_{in}}{T_{out}}\right) \cdot \left(\frac{c_{out}}{c_{in}}\right) = \left(\frac{0.8bar}{1\ bar}\right) \cdot \left(\frac{298.15K}{286.95K}\right) \cdot \left(\frac{150m/s}{10m/s}\right) = 12.47$$

$A_{in} > A_{out}$, it is also proved that this pipe is a nozzle.