A cylinder is enclosed by a piston and filled with 3.2 kg air. The air inside the cylinder is compressed isentropically from the initial state (p_1=1.4 bar, T_1=483K) to final state with pressure p_2=9.8 bar. Please determine

1) the final temperature T_2

2) the work input necessary to compress the air to final state

The heat capacity ratio is κ=1.4.

The air is assumed to be ideal gas and its specific gas constant is R=287 J/kg·K

Solution:

1) Since the process of compression is $p^{1-\kappa} \cdot T^{\kappa}$=const. Then:

$$T_2 = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-\kappa}{\kappa}} = 483\ K \cdot \left(\frac{1.4\ bar}{9.8\ bar}\right)^{\frac{1-1.4}{1.4}} = 842.18\ K$$

2) The work for volume change (here the work input)

$$W_V = -\int_1^2 p \cdot dV = \frac{m \cdot R \cdot T_1}{\kappa - 1} \cdot \left(\left(\frac{V_1}{V_2}\right)^{\kappa-1} - 1\right)$$

Also according to the properties relation of isentropic process:

$$p \cdot V^{\kappa} = const \implies \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\kappa}}$$

Therefore we can rewrite the work for volume change:

$$W_V = -\int_1^2 p \cdot dV = \frac{m \cdot R \cdot T_1}{\kappa - 1} \cdot \left(\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right)$$

$$= \frac{3.2\ kg \cdot 287 J/kg \cdot K \cdot 483K}{1.4 - 1} \cdot \left(\left(\frac{9.8\ bar}{1.4\ bar}\right)^{\frac{1.4-1}{1.4}} - 1\right) = 824.67\ kJ$$