A cylinder is enclosed by a piston and filled with 3.2 kg air. The air inside the cylinder is compressed isentropically from the initial state \((p_1=1.4 \text{ bar, } T_1=483\text{K})\) to final state with pressure \(p_2=9.8 \text{ bar}\). Please determine

1) the final temperature \(T_2\)

2) the work input necessary to compress the air to final state

The heat capacity ratio is \(\kappa=1.4\).

The air is assumed to be ideal gas and its specific gas constant is \(R=287 \text{ J/kgK}\)

Solution:

1) Since the process of compression is \(p^1, \ T^\kappa=\text{const.}\) Then:

\[
T_2 = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-\kappa}{\kappa}} = 483 \text{K} \cdot \left(\frac{1.4 \text{ bar}}{9.8 \text{ bar}}\right)^{\frac{1-1.4}{1.4}} = 842.18 \text{K}
\]

2) The work for volume change (here the work input)

\[
W_V = -\int_1^2 p \cdot dV = \frac{m \cdot R \cdot T_1}{\kappa - 1} \cdot \left(\frac{V_1}{V_2}\right)^{\kappa - 1} - 1
\]

Also according to the properties relation of isentropic process:

\[
p \cdot V^\kappa = \text{const}\Rightarrow \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\kappa}}
\]

Therefore we can rewrite the work for volume change:

\[
W_V = -\int_1^2 p \cdot dV = \frac{m \cdot R \cdot T_1}{\kappa - 1} \cdot \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}} - 1
\]

\[
= \frac{3.2 \text{ kg} \cdot 287\text{J/kgK} \cdot 483\text{K}}{1.4 - 1} \cdot \left(\frac{9.8 \text{ bar}}{1.4 \text{ bar}}\right)^{\frac{1.4 - 1}{1.4}} - 1 = 824.67 \text{kJ}
\]